Impact of the \textit{pdfs} measurements on Higgs production via gluon fusion

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Outline

- The gluon fusion Higgs cross section
  *(what we exactly use to prepare the plots)*

- Experimental uncertainties due to: the *pdfs* $\alpha_s$ $m_t$

- Theoretical uncertainties: scale variations different quark content at NLO
The gluon fusion channel is by far the largest production mechanism and requires our best predictions to discover and to study the Higgs boson.

Although it starts at 1-loop, it is enhanced by the large gluon density of the proton.
Higgs production

\[ \sigma(h_1 + h_2 \rightarrow H + X) = \sum_{a,b} \int_0^1 dx_1 dx_2 f_{a,h_1}(x_1, \mu_F^2) f_{b,h_2}(x_2, \mu_F^2) \times \]
\[ \times \int_0^1 dz \delta \left( z - \frac{\tau_H}{x_1 x_2} \right) \hat{\sigma}_{ab}(z), \]

According to factorization theorems, the hadronic cross section is written as the convolution of the proton pdfs with the partonic cross sections of the relevant parton-parton subprocesses.

The partonic cross sections are computed at a given order in QCD.

A consistent evaluation of the hadronic cross section requires that:
- the pdfs are extracted with the same accuracy;
- the strong coupling is evolved with the same accuracy.
Higgs production at NLO-QCD

Georgi, Glashow, Machacek, Nanopoulos (1978)

In the fermion loop run all possible quarks
Gluon fusion: NLO-QCD corrections in the SM and beyond

\[ \sigma(h_1 + h_2 \rightarrow H + X) = \sum_{a,b} \int_0^1 dx_1 dx_2 \ f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) \times \]

\[ \times \int_0^1 dz \delta\left(z - \frac{\tau_H}{x_1 x_2}\right) \hat{\sigma}_{ab}(z) \]

\[ \mathcal{M} = \begin{array}{c}
\quad +
\end{array} \]

\[ \hat{\sigma}_{ab}(z) = \sigma^{(0)} z \ G_{ab}(z) \]

\[ \sigma^{(0)} = \frac{G_\mu \alpha_s^2(\mu_R^2)}{128 \sqrt{2} \pi} \left| \sum_{i=0,1/2} \lambda_i \left(A^2 \right)^{1-2i} T(R_i) G_i^{(1l)} \right|^2 \]
Gluon fusion: NLO-QCD corrections in the SM and beyond

\[ G_i^{(2l)} = \lambda_i \left( \frac{A^2}{m_0^2} \right)^{1-2i} T(R_i) \left( C(R_i) G_i^{(2l,C_R)}(x_i) + C_A G_i^{(2l,C_A)}(x_i) \right) \times \left( \sum_{j=0,1/2}^{1} \lambda_j \left( \frac{A^2}{m_0^2} \right)^{1-2j} T(R_j) G_j^{(1l)} \right)^{-1} + h.c. \]
Gluon fusion: NLO-QCD corrections in the SM and beyond

Baur, Glover    Ellis, Hincliff, Soldate, van der Bij    Bonciani Degrassi, AV

also all the function $A_{ij}$ expressed in terms of HPLs

$$\mathcal{R}_{gg} = \frac{1}{z(1-z)} \int_0^1 \frac{dv}{v(1-v)} \left\{ \frac{8 z^4 \left| A_{gg}(\hat{s}, \hat{t}, \hat{u}) \right|^2}{\left| \sum_{j=0,1/2} \lambda_j \left( \frac{A^2}{m_0^2} \right)^{1-2j} T(R_j) G_j^{(11)} \right|^2} - (1 - z + z^2)^2 \right\}$$

$$\mathcal{R}_{q\bar{q}} = \frac{128}{27} \frac{z (1 - z) \left| A_{q\bar{q}}(\hat{s}, \hat{t}, \hat{u}) \right|^2}{\left| \sum_{j=0,1/2} \lambda_j \left( \frac{A^2}{m_0^2} \right)^{1-2j} T(R_j) G_j^{(11)} \right|^2}$$

$$\mathcal{R}_{qg} = C_F \int_0^1 \frac{dv}{(1-v)} \left\{ \frac{1 + (1-z)^2 v^2}{[1 - (1-z)v]^2} \left[ \frac{2 z \left| A_{qg}(\hat{s}, \hat{t}, \hat{u}) \right|^2}{\left| \sum_{j=0,1/2} \lambda_j \left( \frac{A^2}{m_0^2} \right)^{1-2j} T(R_j) G_j^{(11)} \right|^2} - \frac{1 + (1-z)^2}{2z} \right] \right\} + \frac{1}{2} C_F z$$
QCD K-factor and the large-mt limit

large-mt limit: the top triangle loop shrinks to a pointlike interaction vertex

if we consider only the top quark (or only one scalar) in the loop, then the K-factor in the large-mt limit is already a good approximation of the exact K-factor

but we have to include at least also the bottom!
if we include QCD corrections to the light particles in the loop
the K-factor in the exact calculation may differ from the approximated one

also at NLO non trivial interferences between
the top and bottom (and charm, strange,...) amplitudes
Alike the bottom, also the charm quark has a negative effect on the cross section because of a negative interference effect between top and charm amplitudes.

All these ingredients are available since 1995 in HIGLU (very good numerical agreement of our code with HIGLU)!
$m_t$ dependence

For light Higgs, dependence on the precise value of the top mass below 0.5%

Around and above the $t\bar{t}$-bar threshold, non trivial dependence
Higgs production beyond NLO-QCD

- NNLO-QCD results in the $m_t \to \infty$ limit (+15%)

- finite $m_t$ effects at NNLO-QCD (~0.5%)
  Marzani, Ball, Del Duca, Forte, Vicini (2008), Harlander, Ozeren (2009)

- soft-gluon resummation at NNLL-QCD (+6%)
  Catani, De Florian, Grazzini, Nason (2003)

- inclusion of leading NNNLO-QCD contributions (+5%)
  Moch, Vogt (2005)

- full NLO-EW corrections (+4-7%)

Further increase, beyond NLO-QCD, of the total cross section:
  +25-30% of the Born

Stability against renormalization/factorization scale variation

Good accuracy of the partonic cross section

→ In this talk I will mostly discuss NLO-QCD results where pdfs can be compared

extraction from the data of pdfs with NNLO-QCD accuracy would require the use of full set (unavailable) of NNLO-QCD calculations
Higgs production at NLO-QCD

The gluon subprocesses (both real and virtual) give the largest contribution to the total cross-section.

Need to reduce the uncertainty on the gluon pdf.
values of $x$ relevant to produce a final state with invariant mass of 100 GeV in the central rapidity interval $|y_H| \leq 2$

$y = \frac{M}{Q}$

$Q = M$

$LHC \quad [0.00096, 0.052]$

$Tevatron \quad [0.0069, 0.376]$
Kinematics

The small-$x$ region plays a minor role in the evaluation of the inclusive cross section (dominated by lowest-order threshold kinematics).

Cross section dominated by the lowest order partonic threshold kinematics
Large contribution due to soft gluon emission at the threshold
One small-$x$ value in one pdf requires a large-$x$ value in the other, but the steep fall of the gluon density suppresses these contributions

The uncertainties associated to the small-$x$ region do not affect the total cross section

Differences of the gluon density in the large-$x$ region, due e.g. to different treatment of jet data, may appear at intermediate $x$ because normalization/momentum sum rules constraints
Higgs production at NLO-QCD: pdfs uncertainties

The uncertainty due to the experimental errors of the data, from which the pdfs are extracted, is parametrized in different ways:

- Montecarlo replicas
- Hessian method

The corresponding definitions to compute the standard deviation associated to an observable $\mathcal{F}$ is

$$\sigma_{\mathcal{F}} = \left( \frac{1}{N_{\text{set}} - 1} \sum_{k=1}^{N_{\text{set}}} \left( \mathcal{F}[\{q^{(k)}\}] - \langle \mathcal{F}[\{q\}] \rangle \right)^2 \right)^{1/2}$$

$$\sigma_{\text{hepdata}} = \frac{1}{2C_{90}} \left( \sum_{k=1}^{N_{\text{set}}/2} \left( \mathcal{F}[\{q^{(2k-1)}\}] - \mathcal{F}[\{q^{(2k)}\}] \right)^2 \right)^{1/2}$$

The factor $C_{90}$ is not necessary any more for MSTW2008, where different sets are provided, which parametrize the uncertainties at 68% and at 90% C.L.

In our exercise $\mathcal{F}$ is: the inclusive Higgs production cross section at NLO-QCD the parton densities
Effects of Altarelli-Parisi evolution

the $x$ relevant for the gluon fusion are well measured at HERA
Comparison of $pdf$s

CTEQ6.6

DIS data, DY data, jets at colliders (not included: Tevatron run II, NuTeV, CHORUS data)
the data are fitted with a functional form fixed a priori
propagation of experimental errors to the $pdf$s described using the Hessian method

MSTW2008

DIS data, DY data, jets at colliders
the data are fitted with a functional form fixed a priori
propagation of experimental errors to the $pdf$s described using the Hessian method

NNPDF1.2

only DIS scattering data (NC and CC)
MC replicas of the data are generated within the experimental error
a Neural Network learns from the replicas a possible parametrization of the $pdf$s
the final set of $pdf$s provides a sampling of the $pdf$s parameter space
and allows to measure, in a statistically meaningful way, the spread of any observable
not yet full treatment of heavy quarks
CTEQ gluon densities
Higgs production at NLO-QCD: changes using CTEQ pdfs

larger changes for large MH
CTEQ cross sections

pp → Hx NLO cross section ratio to CTEQ 6.1 cross section at $\sqrt{s} = 14$ TeV with experimental uncertainty error bands
MSTW gluon densities
Higgs production at NLO-QCD: changes using M(R)ST(W) sets

larger changes for small MH
MSTW cross sections

pp → Hx NLO cross section ratio to MRST 2001 E cross section
at $\sqrt{s} = 14$ TeV with experimental uncertainty error bands
Comparison of *pdfs*: historical changes

**CTEQ**

Heavy quark mass effect were absent in 6.1 and have been introduced in 6.5/6.6.

New treatment of the strange quark density

Important changes for up, down and gluon

**MSTW**

New data: neutrino str.fun., CCFR and NuTeV, Z rapidity, Run II inclusive jet prod., HERA.

GM-VFNS to treat HQ effects (first introduced in 2006), new richer parametrization

change in the alpha_s value (reduced from 2006 to 2008) yields larger gluon at small-x
Comparison of *pdfs*: latest released sets

Size of uncertainties similar for CTEQ and NNPDF

Smaller by a factor 2 for MSTW

In the region constrained by HERA data

NNPDF agrees with CTEQ and MSTW
MSTW2008 and CTEQ6.6 do not overlap for MH<150 GeV

NNPDF1.0 agrees with CTEQ6.6 for MH>200 GeV with MSTW over entire range

$$\alpha_s(m_Z) =$$

CTEQ6.6 0.118
NNPDF1.0 0.119
MSTW2008 0.120
Higgs total cross section at NLO-QCD: Tevatron

Similar uncertainties quoted by the 3 collaborations

For light MH, significant (up to 3σ) discrepancy (up to 9%) in the central values
NLO xsecs: current pdfs uncertainties

LHC 8 TeV

MSTW2008 and CTEQ6.6 do not overlap for MH<500 GeV

NNPDF1.0 overlaps with CTEQ6.6 for MH<120 GeV with MSTW for MH>200 GeV

$\alpha_s(m_Z) = \begin{cases} 
\text{CTEQ6.6} & 0.118 \\
\text{NNPDF1.0} & 0.119 \\
\text{MSTW2008} & 0.120 
\end{cases}$
MSTW2008 and CTEQ6.6 do not overlap for $MH<500$ GeV

NNPDF1.0 agrees with CTEQ6.6 for $MH<180$ GeV with MSTW for $MH>200$ GeV

$$\alpha_s(m_Z) = \begin{array}{c}
\text{CTEQ6.6} & 0.118 \\
\text{NNPDF1.0} & 0.119 \\
\text{MSTW2008} & 0.120
\end{array}$$
Similar uncertainties (worst agreement w.r.t. Tevatron) quoted by the 3 collaborations

Significant (up to $3\sigma$) discrepancy (6%) in the central values

The minimum of uncertainty for a range of Higgs masses corresponding to a range of $x$ where HERA data play a major role on the gluon determination
Possible origin of the discrepancy

Differences in the gluon density can account for part of the discrepancy but the 3 collaborations use different values for $\alpha_s(m_Z)$ and the cross section has a strong dependence on this coupling

$$\sigma_{tot} = \alpha_s^2 \sigma_0 + \alpha_s^3 \sigma_1 + \ldots \quad \sigma_0 \sim \sigma_1$$

$$\alpha_s(m_Z) = 0.1176(20)$$

Naively, a change of $\alpha_s$ by $1\sigma$ implies a change of the cross section of 3-5 %

$$\frac{2}{117} \sim 0.017 \quad \text{for constant } pdfs \quad 0.034 < \delta\sigma_{tot} < 0.051$$

but parton densities and strong coupling should be consistent with each other

it is necessary to study the role of the uncertainties on $\alpha_s$ in the global fit of the $pdfs$
The strong coupling can be determined from observables that do not depend on the $pdf$s and then it can be used, with its error, to evaluate the impact of this parameter on the cross section but the strong coupling is very precisely measured in processes that involve the $pdf$s it is necessary to evaluate to which extent $\alpha_s$ and the $pdf$s are correlated (e.g. gluon density is directly proportional to $\alpha_s$)
$\alpha_s$ and gluon density in CTEQ6.6

Gluon at $\mu = 100$ GeV

gluon density obtained from a global fit with different values of $\alpha_s(M_Z)$

$$0.110 \leq \alpha_s(m_Z) \leq 0.126$$

compared with the uncertainty band of the best fit with central $\alpha_s$

the vertical lines mark the x range to produce an object of 100 GeV with $|y|<2$
**$\alpha_s$ dependence, MSTW**

For MSTW, the uncertainty on the *pdfs* and on $\alpha_s$ are correlated

$$\alpha_s \in [\alpha_s^0 - 1\sigma, \alpha_s^0 + 1\sigma]$$

$$\alpha_s^0 \equiv \alpha_s(m_Z) = 0.1202^{+0.0012}_{-0.0015}$$

For each of the 5 values:

$$\alpha_s^0 - 1\sigma, \quad \alpha_s^0 - 0.5\sigma, \quad \alpha_s^0, \quad \alpha_s^0 + 0.5\sigma, \quad \alpha_s^0 + 1\sigma$$

there are 40 pdf sets

Some *pdfs* spreads are much smaller that the central-value spread
\( \alpha_s \) dependence, MSTW

For each of the 5 values compute the pdf spread (not necessarily symmetric)

\[
(\Delta F_{PDF}^{\alpha_s})_+ = \frac{1}{\sqrt{n}} \sum_{k=1}^{n} \{ \max \left[ F^{\alpha_s}(S_k^+) - F^{\alpha_s}(S_0), 0 \right] \}^2,
\]

\[
(\Delta F_{PDF}^{\alpha_s})_- = \frac{1}{\sqrt{n}} \sum_{k=1}^{n} \{ \max \left[ F^{\alpha_s}(S_0) - F^{\alpha_s}(S_k^-), 0 \right] \}^2,
\]

The (pdf+alpha_s) spread is obtained as follows

\[
(\Delta F_{PDF+\alpha_s})_+ = \max_{\alpha_s} (\{ F^{\alpha_s}(S_0) + (\Delta F_{PDF}^{\alpha_s})_+ \}) - F^{\alpha_s}_0(S_0),
\]
\[
(\Delta F_{PDF+\alpha_s})_- = F^{\alpha_s}_0(S_0) - \min_{\alpha_s} (\{ F^{\alpha_s}(S_0) - (\Delta F_{PDF}^{\alpha_s})_- \}),
\]

Some pdfs spreads are much smaller that the central-value spread
This recipe is quite conservative
$\alpha_s$ dependence, MSTW

\[ \frac{\delta \sigma}{\sigma} \]

\[ m_H \text{ (GeV)} \]

\[ \sigma_{\text{tot}} \text{ (pb)} \]

\[ \alpha_s + \text{pdf} \pm 68\% \]

\[ \alpha_s + \text{pdf} \pm 90\% \]

\[ \text{MSTW2008nlo} \]

\[ \text{LHC 14 TeV} \]
dependence, CTEQ

Series of global fits compatible with existing data using as reference value for $\alpha_s$ values within the uncertainty band of $\alpha_s(m_Z)$

Every choice of $\alpha_s(m_Z)$ yields a different gluon density

CTEQ assumes that the uncertainties due to the pdf and and those due to $\alpha_s(m_Z)$ are not correlated and can be combined in quadrature

The central values are instead strongly correlated
$\alpha_s$ dependence, comparison CTEQ6.6 vs MSTW2008

The assumptions are different
the overall size of the combined effect is similar
the estimate at small $m_H$ is different

These larger uncertainty bands reduce the significance of the discrepancy of the two sets

$\alpha_s = 0.118 \pm 2\sigma$

$\sqrt{(\alpha_s)^2 + (pdf)^2}$

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Scale dependence (NNLO-QCD):

Independent variation of factorization and renormalization scales.

NNLO-QCD contributions in the infinite top mass limit added to the exact NLO-QCD

| Independent combinations of relative theoretical uncertainties on pp → Hx cross section at √s = 1.96 TeV at NNLO |
|---|---|---|---|---|---|---|---|
| | | | | | | | |

| Independent combinations of relative theoretical uncertainties on pp → Hx cross section at √s = 8 TeV at NNLO |
|---|---|---|---|---|---|---|---|
| | | | | | | | |

| Independent combinations of relative theoretical uncertainties on pp → Hx cross section at √s = 14 TeV at NNLO |
|---|---|---|---|---|---|---|---|
| | | | | | | | |

The lines show the 4 combinations in units MH:

\[(\mu_{\text{ren}}, \mu_{\text{fac}}) = (0.5, 0.5), (0.5, 2), (2, 0.5), (2, 2)\]
Scale dependence: Tevatron

\[ 
MSTW 2008 \text{ lo} \\
MSTW 2008 \text{ nlo} \\
MSTW 2008 \text{ nnlo} 
\]

with theoretical uncertainty error bands

The bands are large and have partial overlap

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Scale dependence: LHC 14 TeV

The bands are thinner than at the Tevatron but have smaller overlap.
Conclusions

- The experimental uncertainty on the pdfs affects the gluon fusion cross section at the few per cent level (the relevant x are well measured at HERA)
- Estimate of the size of the uncertainty similar for CTEQ, MSTW and NNPDF
- Not negligible differences of the central values between CTEQ6.6, MSTW2008 at NLO-QCD
- The role of alpha_s can partially reduce the significance of the discrepancy, enlarging the error bands
- Is the difference of the xsec, at NLO-QCD, part of the pdf uncertainty?
- The size of the uncertainty bands due to variation of the scales are at the level of 6-15% at NNLO-QCD; non trivial shape of the bands
- The accurate prediction of the gluon fusion total cross section requires, at least, the use of exact NLO-QCD predictions with all quark flavors